PRE-CALCULUS TRIG APPLICATIONS UNIT
Simplifying Trigonometric Expressions

What is an Identity?

What is it used for?

The Reciprocal Identities:

$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

$\csc\theta =$ $\sec\theta =$ $ctn\,\theta =$

The Quotient Identities:

$\tan\theta =$ $ctn\,\theta =$

The Negative Angle Identities:

The sine is an odd function (sym w/r to the origin), so $\sin(-\theta) =$
The cosine is an even function (sym w/r to the y-axis), so $\cos(-\theta) =$

The Pythagorean Identities:

The most famous one: Other forms:

The other two:

Simplifying Trigonometric Expressions:

Simplify each into a single trig function with no fractions:

$\csc(\alpha)\tan(\alpha) =$

$\frac{\sec(t)}{\tan(t)} =$

$\frac{\sec(t) - \cos(t)}{\sin(t)} =$

$\frac{\cos^2(\theta) - 1}{\csc(\theta)} =$

Homework for Simplifying Trigonometric Expressions

Simplify each into a single trig function with no fractions!

$$\frac{sec(\beta)}{csc(\beta)}$$

$$\frac{ctn(x)}{csc(x) - sin(x)}$$

$$\left(1 + tan^2(x)\right)(cos^2(x))$$

$$(ctn^2(x) + 1)(cos^2(x))$$

$$\frac{1 + tan(\alpha)}{1 + ctn(\alpha)}$$

$$\frac{1 + cos(\theta)}{1 + sec(\theta)}$$

$$\frac{1 + tan^2(x)}{sin^2(x)}$$

$$\frac{cos^2(x)}{sin(x) - 1}$$

$$\frac{\left(1 - cos(x)\right)(1 + cos(x))}{sin^2(x)}$$

$$\frac{1}{1 + tan^2(\theta)} + \frac{1}{1 + ctn^2(x)}$$

PRE-CALCULUS TRIG APPLICATIONS UNIT
Proving Trigonometric Identities

What does it mean to "Prove" an Identity?

Tips to Help Prove Trigonometric Identities:

1. **Work the more complicated side.**

2. **Scan for Identities**

3. **When in doubt . . .**

4. **Use Algebra...**

Examples:

$$\frac{\cos^2(x)}{1+\sin(x)} = 1 - \sin(x)$$

$$\frac{1+ctn(\theta)}{\csc(\theta)} = \sin(\theta) + \cos(\theta)$$

$$\frac{\csc^2(x) - \sin^2(x)}{\csc(x) + \sin(x)} = \cos(x)\cot(x)$$

Homework for Proving Trigonometric Identities

Prove the following Identities:

$$sec(a) - cos(a) = sin(a)tan(a)$$

$$\frac{1 + tan^2(b)}{tan^2(b)} = csc^2(b)$$

$$\frac{(1 - cos(y))(1 + cos(y))}{sin(y)} = sin(y)$$

$$\frac{1}{1 + ctn^2(\theta)} + \frac{1}{1 + tan^2(\theta)} = 1$$

$$\frac{tan\ x - sin\ x\ cos\ x}{sin^2 x} = tan\ x$$

$$\frac{csc^2(\delta) - 1}{csc^2(\delta) - csc(\delta)} = 1 + sin(\delta)$$

$$\frac{1 + cos(u)}{sin(u)} = \frac{sin(u)}{1 - cos(u)}$$

$$\frac{sin^4(x) - cos^4(x)}{sin(x) - cos(x)} = sin(x) + cos(x)$$

PRE-CALCULUS TRIG APPLICATIONS UNIT
More with Trig Identities

Simplify:

$$\frac{\csc x}{\text{ctn}\, x + \tan x}$$

$$\frac{\cos x \ \tan x}{\csc x}$$

Verify:

$$1 - 2\sin^2 r + \sin^4 r = \cos^4 r$$

$$\tan u + \frac{\cos u}{1+\sin u} = \sec u$$

$$\frac{\tan x - \sin x \cos x}{\sin^2 x} = \tan x$$

$$\frac{\sec x - 1}{\sec x + 1} + \frac{\cos x - 1}{\cos x + 1} = 0$$

$$\frac{\csc x}{\operatorname{ctn} x + \tan x} = \cos x$$

$$\frac{\tan x + \sec x}{\sec x - \cos x + \tan x} = \csc x$$

PRE-CALCULUS TRIG APPLICATIONS UNIT
Solving Trigonometric Equations

Solving equations for values between 0 and 2π.

$$2sin(\theta) - 1 = 0$$

$$2cos(x) - \sqrt{3} = 0$$

Solving trigonmetric equations for all values.

$$sin^2(x) - sin(x) - 2 = 0$$

$$2sin(3\theta) = 1$$

$$cos\left(\frac{\pi}{4}\theta\right) = -1$$

Find the first two positive values for x:

$$7sin(6t) = 2$$

The height of a rider on the Ferris Wheel at Cedar Point can be determined by the equation: $h(t) = -67.5cos\left(\frac{\pi}{15}t\right) + 69.5$. How long is the rider more than 100 meters above the ground?

Homework for Solving Trigonometric Equations

Solve for all values between *0 and 2π*:

$$2sin(\theta) = -\sqrt{2}$$

$$sin(\theta) = 0$$

Find all solutions (Use no calculator!):

$$2cos(\theta) = \sqrt{2}$$

$$2cos(\theta) = 1$$

$$2sin(3\theta) = 1$$

$$2sin(3\theta) = -\sqrt{2}$$

$$sin\left(\frac{\pi}{3}\theta\right) = -1$$

$$2cos\left(\frac{\pi}{4}\theta\right) = \sqrt{3}$$

Use a calculator to find the first two positive values for x:

$$5cos(3x) = -3$$

$$3sin\left(\frac{\pi}{4}x\right) = 2$$

PRE-CALCULUS TRIG APPLICATIONS UNIT
More on Solving Trigonometric Equations

Factor and solve the following:

$2x^2 - x = 0$

$x^2 - x - 2 = 0$

Now, try factoring and solving these for θ *between* 0 *and* 2π:

$2sin^2(\theta) - sin(\theta) = 0$

$cos^2(x) - cos(x) - 2 = 0$

Using Identities to help solve:

$2sin^2(t) - cos(t) = 1 \quad \text{for all solutions with } 0 \le t \le 2\pi$

$tan(x) = 3sin(x) \quad \text{for all solutions with } 0 \le x \le 2\pi$ **(You will need a calculator to finish part of the problem)**

Homework for More on Solving Trigonometric Equations

Find all solutions on the interval $[0, 2\pi]$

$2sin\left(\frac{\pi}{4}x\right) = 1$

$2cos(2t) = -\sqrt{3}$

$sec(\theta) = 2cos(\theta)$

$2sin^2(t) + 3sin(t) + 1 = 0$

$sec(x)sin(x) - 2sin(x) = 0$

$sin^2(x) = \frac{1}{4}$

$tan^3(x) = 3tan(x)$

$4cos^2(x) - 4 = 15cos(x)$

PRE-CALCULUS TRIG APPLICATIONS UNIT
Sum and Difference Formulas

The Sum and Difference Formulas:

$\boldsymbol{sin(\alpha + \beta) =}$ $\boldsymbol{sin(\alpha - \beta) =}$

$\boldsymbol{cos(\alpha + \beta) =}$ $\boldsymbol{cos(\alpha - \beta) =}$

Using the Formulas:

Find the Exact value of $\boldsymbol{cos(75°) =}$

Rewrite: $\boldsymbol{sin\left(x - \frac{\pi}{4}\right) =}$

Solve: $\boldsymbol{sin(x)sin(2x) + cos(x)cos(2x) = \frac{\sqrt{3}}{2}}$

Given: $\boldsymbol{sin(a) = \frac{3}{5}\ and\ cos(b) = \frac{5}{13}\ with\ both\ a\ and\ b\ in\ the\ first\ quadrant:}$

Find: $\boldsymbol{sin(a + b)}$ $\boldsymbol{cos(a - b)}$

Homework for Sum and Difference Formulas

Given: $sin(a) = \frac{2}{5}$ $and\ cos(b) = \frac{5}{6}$ $with\ both\ a\ and\ b\ in\ the\ first\ quadrant$:

Find: $sin(a + b) =$ $cos(a - b)$

Find the Exact values:

$sin(195°) = sin(\quad + \quad)$ $cos\left(\frac{7\pi}{12}\right) = cos(\quad + \quad)$

Expand and Simplify:

$sin\left(x + \frac{11\pi}{6}\right) =$

$csc\left(\frac{\pi}{2} - t\right) =$

Solve for x:

$$cos(2x)cos(x) + sin(2x)sin(x) = 1$$

PRE-CALCULUS TRIG APPLICATIONS UNIT
Double Angle Identities

Double Angle Identities:

$sin(2\alpha) =$ $cos(2\alpha) =$

Using the Double Angle Identities:

$If\ sin(\theta) = \frac{3}{5}\ and\ \theta\ is\ in\ the\ 2nd\ quadrant, find\ the\ exact\ values\ of$:

$sin(2\theta)$ $cos(2\theta)$

Simplify the expressions:

$2cos^2(12°) - 1$ $8sin(3x)cos(3x)$

Prove: $sec(2\alpha) = \frac{sec^2(\alpha)}{2 - sec^2(\alpha)}$

Use an identity to find the exact value of: $cos^2(75°) - sin^2(75°)$

Solve: $cos(2t) = cos(t)\ \ for\ all\ solutions\ with\ 0 \leq t \leq 2\pi$

Homework for Double Angle Identities

$If\ sin(x) = 1/8\ \ and\ x\ is\ in\ quadrant\ I, then\ find\ the\ exact\ values\ for:$

$sin(2x)$ $cos(2x)$

Simplify each expression:

$cos^2(28°) - sin^2(28°)$ $1 - 2sin^2(17°)$

$cos^2(9x) - sin^2(9x)$ $4sin(8x)cos(8x)$

Solve for all solutions on the interval $[0, 2\pi]$

$6sin(2t) + 9sin(t) = 0$ $sin(2t) = cos(t)$

Prove the identities below:

$(sin\ t - cos\ t)^2 = 1 - sin(2t)$ $sin(2x) = \frac{2tan(x)}{1 + tan^2(x)}$

PRE-CALCULUS TRIG APPLICATIONS UNIT
Half Angle Identities

Half Angle Identities:

$\cos\left(\frac{\theta}{2}\right) =$

$\sin\left(\frac{\theta}{2}\right) =$

Using the Half Angle Identities:

Find the exact value for the $\cos(15°)$

$If\ \sin(x) = 1/5\ and\ x\ is\ in\ quadrant\ I, then\ find\ the\ exact\ values\ for:$

$\sin\left(\frac{x}{2}\right) =$

Power Reduction Identities:

$\cos^2(\alpha) =$

$\sin^2(\alpha) =$

Rewrite $\cos^4(x)$ **without any powers.**

Homework for Half Angle Identities

Use Half Angle Identities to find the Exact values of:

$sin(75°) =$

$cos\left(\frac{\pi}{12}\right) =$

Use the Power Reduction Identities to rewrite without exponents.

$cos^2(5x) =$

$sin^4(8x) =$

$If\ csc(x) = 7\ and\ 90° < x < 180°, then\ find\ exact\ values\ for:$

$sin\left(\frac{x}{2}\right)$

$cos\left(\frac{x}{2}\right)$

$tan\left(\frac{x}{2}\right)$