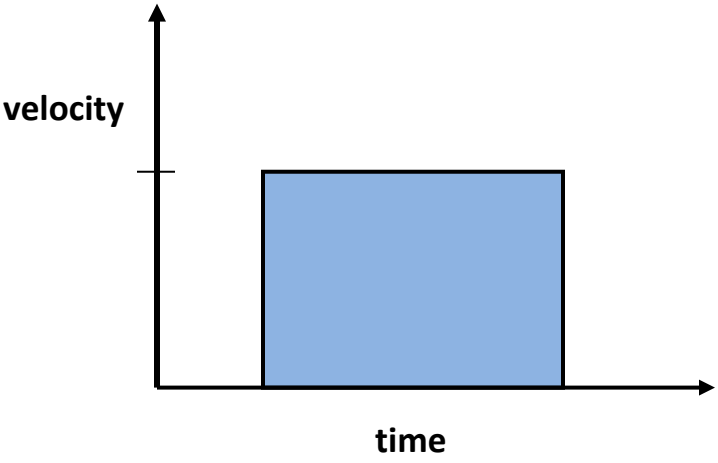
CALCULUS CHAPTER 5 NOTES

SECTION 5-1 AREAS UNDER CURVES

Integral Calculus:

Integration:

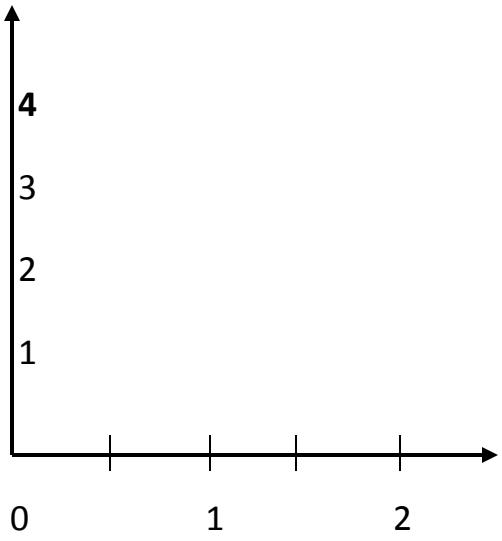
Finding distance traveled:

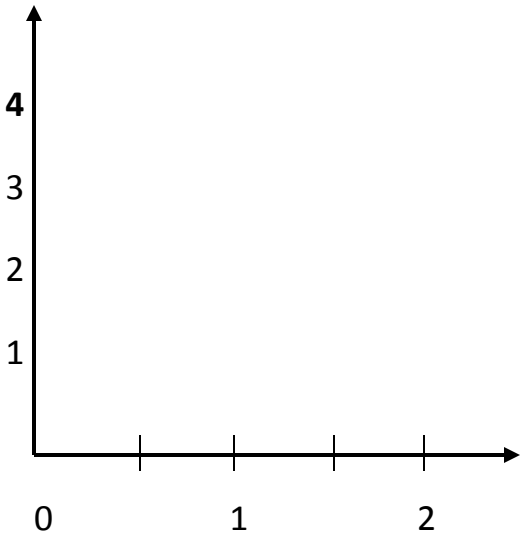


What about curves?

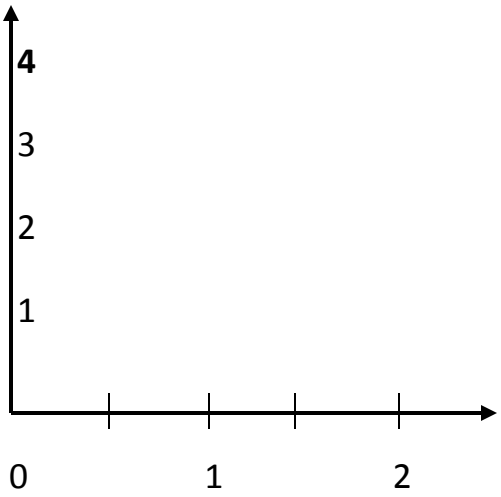
Find the area under the curve $y = x^2$ from $0 \leq x \leq 2$ using 4 subintervals.

LRAM **RRAM**





MRAM



Distance traveled Upstream: You are walking along the bank of a tidal river watching the incoming tide carry a bottle upstream. You record the velocity of the flow every 5 minutes for an hour. You record the results in the table below. About how far upstream (distance) does the bottle travel during that hour? Use a.) LRAM, b.) RRAM and c.) the Trapezoidal Rule.

Time (min)	0	5	10	15	20	25	30	35	40	45	50	55	60
Vel (m/sec)	1	1.2	1.7	2.0	1.8	1.6	1.4	1.2	1.0	1.8	1.5	1.2	0

a.)

b.)

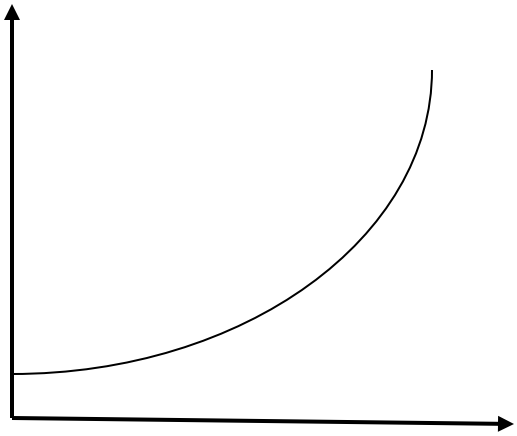
c.)

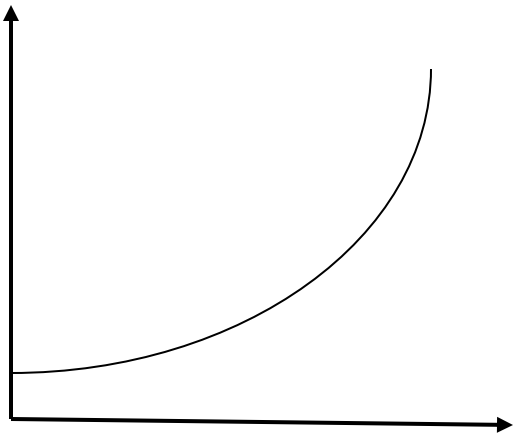
ASSIGNMENT: Page 254 – 255 # 1, 2, 10, 12

CALCULUS CHAPTER 5 NOTES

SECTION 5-2 Reimann Sums

Reimann Sum:





Sigma Notation:

$$\sum_{k=1}^{6} k^2 + 1$$

Definition of an Antiderivative:

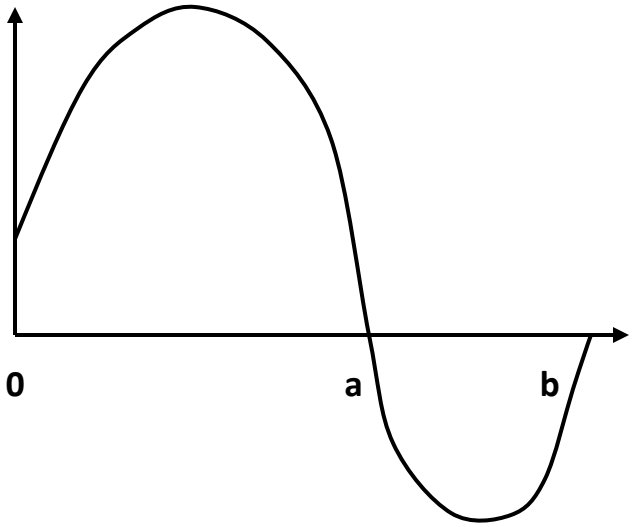
$$\lim_{n\to\infty} \sum_{k=1}^{n} f(c_k)\,\Delta x = \int_a^b f(x)\,dx$$

Example: Convert to Integrals:

$$\lim_{n\to\infty} \sum_{k=1}^{n} (4(m_k)^3 - 6(m_k) - 7)\Delta x$$

AREA UNDER A CURVE

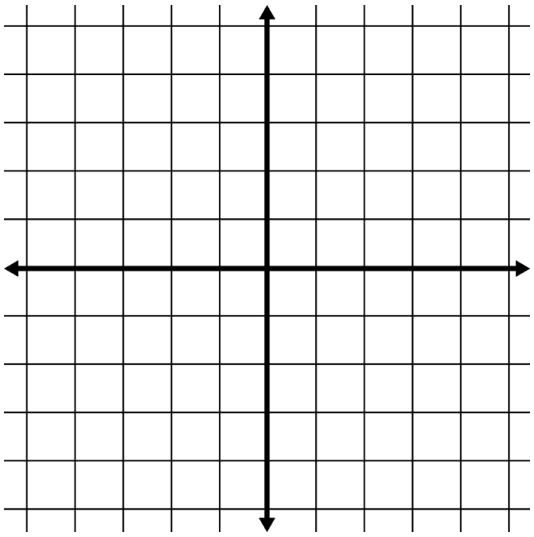
Recall: An antiderivative represents:



Area from 0 to a: $\int$ **Area from a to b:** $\int$ **Area from 0 to b:** $\int$

Evaluating integrals of Constant functions:

$$\int_{-2}^{5} -2\ dx$$



ASSIGNMENT: Page 267 Quick review #1, 2, 4 - 8; Exercises #1-12, 29-32

CALCULUS CHAPTER 5 NOTES

SECTION 5-3 Definite Integrals

What is a Definite Integral?

An Integral is . . .

<u>Rules for Definite Integrals</u>:

$$\mathbf{1.}\ \boldsymbol{Order\ if\ Integration}\text{: } \int_{b}^{a} f(x)\,dx =$$

$$\mathbf{2.}\ \boldsymbol{Zero}\text{: } \int_{a}^{a} f(x)\,dx =$$

$$\mathbf{3.}\boldsymbol{Constants}\text{: } \int_{a}^{b} k \cdot f(x)\,dx =$$

$$\mathbf{4.}\ \boldsymbol{Sum\ and\ Difference}\text{: } \int_{a}^{b} (f(x) \pm g(x))\,dx =$$

$$\mathbf{5.}\ \boldsymbol{Additivity}\text{: } \int_{a}^{b} f(x)\,dx + \int_{b}^{c} f(x)\,dx =$$

Suppose:

$$\int_{-1}^{1} f(x)\,dx = 5,\ \int_{1}^{4} f(x)\,dx = -2, and\ \int_{-1}^{1} h(x)\,dx = 7$$

$$a.)\ \int_{41}^{1} f(x)\,dx = \qquad b.)\ \int_{-1}^{4} f(x)\,dx = \qquad c.)\ \int_{-1}^{1} [2f(x) + 3h(x)]\,dx =$$

$$d.)\ \int_{0}^{1} f(x)\,dx = \qquad e.)\ \int_{-2}^{2} h(x)\,dx = \qquad f.)\ \int_{-1}^{4} [f(x) + h(x)]dx =$$

2nd Fundamental Theorem of Calculus: **If *F* is the antiderivative of *f* (a continuous function on [a, b]), then:**

$$\int_a^b f(x)\,dx =$$

Example:

$$\int_{-1}^{1} (x-3)dx =$$

$$\int_0^{\pi} \sin x\ \ dx =$$

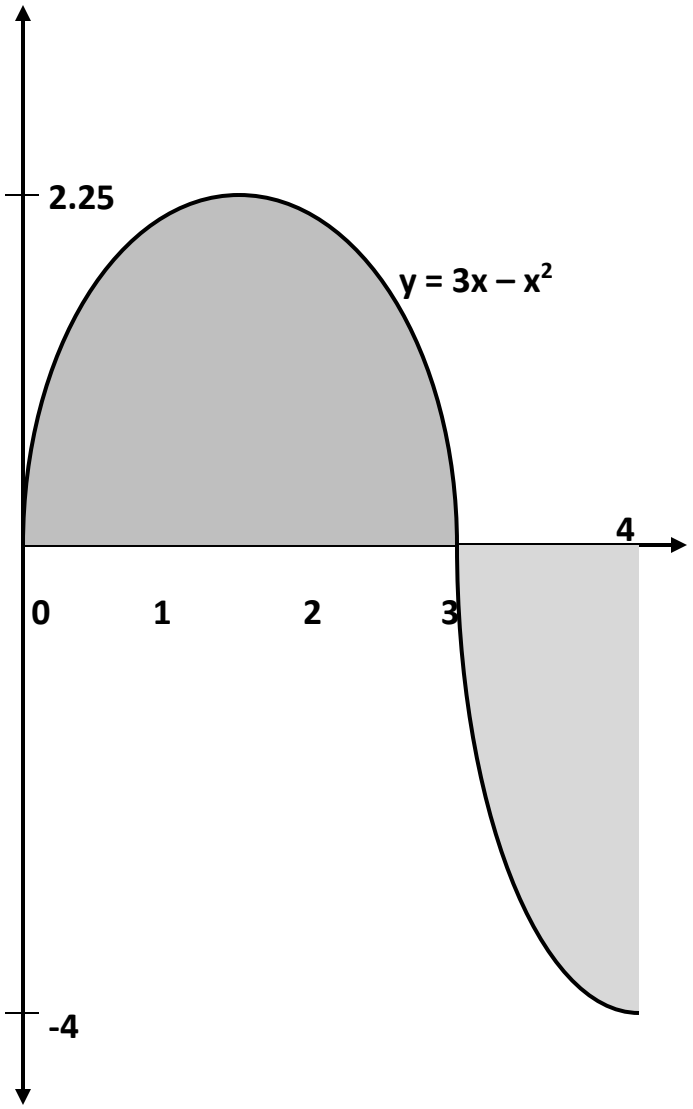
$$\int_e^1 \frac{1}{x}\,dx =$$

ASSIGNMENT: Page 274 – 275 #1 - 12

CALCULUS CHAPTER 5 NOTES

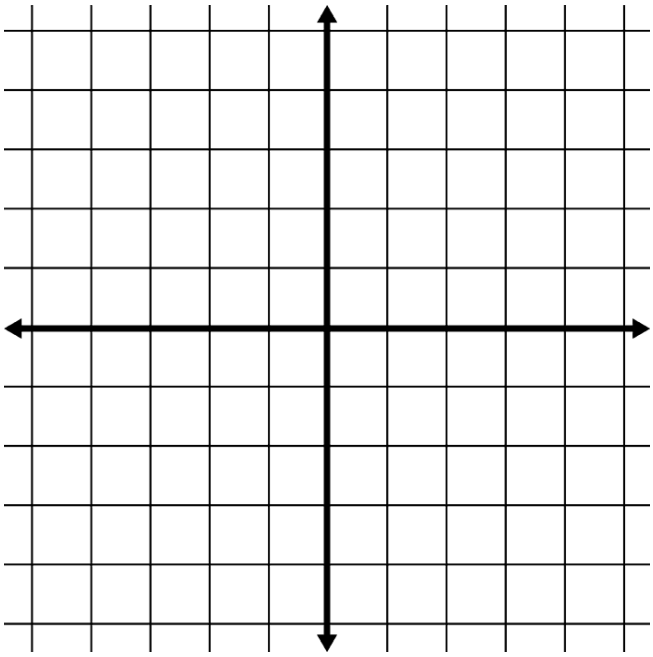
SECTION 5-3 (Day 2) Average Value

Finding the Total Shaded Area:



<u>The Average (Mean) Value of a Function:</u> If f is integrable on [a, b], its average value on [a, b] is:

Example: Find the Average Value of $f(x) = 4 - x^2$ on [0, 3].



Now, find the x-value on [0, 3] where the f(x) is at its average.

ASSIGNMENT: Page 275 #13 – 16, 20, 21 (crosses at x = 2), 22 (crosses at x = 1)

CALCULUS CHAPTER 5 NOTES

SECTION 5-4 1st Fundamental Thm of Calculus

1st Fundamental Theorem of Calculus: If f is continuous on [a, b], then the function

$$F(x) = \int_a^x f(t)\,dt$$

$$\textbf{has a derivative at every point } x \textbf{ in } [a, b] \textbf{ and}:$$

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)dt =$$

What does this mean?

Examples:

$$\frac{d}{dx}\int_{-\pi}^{x} \cos t\,dt = \qquad\qquad \frac{d}{dx}\int_0^x \frac{1}{1+t^2}\,dt =$$

$$\frac{d}{dx}\int_x^3 4t \sin t\,dt =$$

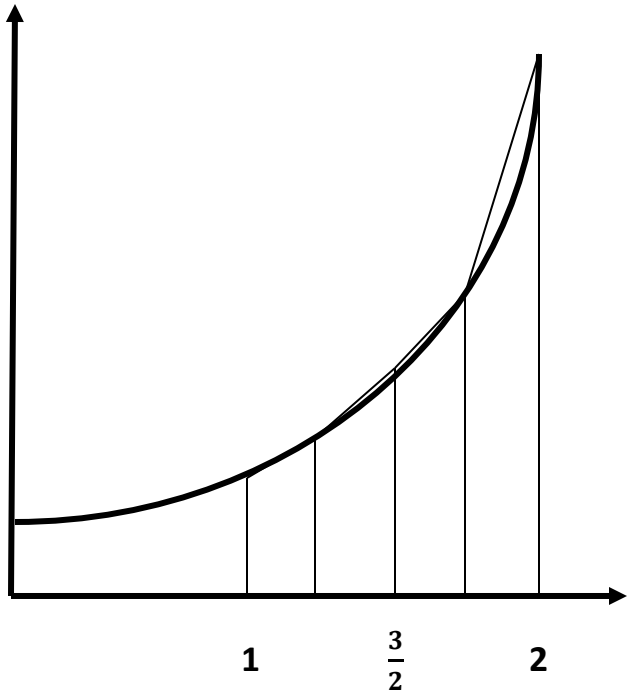
$$\frac{d}{dx}\int_1^{x^2} \cos t\,dt = \cos x^2 \cdot \frac{d}{dx}(x^2)\ (\textbf{The Chain Rule})$$

ASSIGNMENT: Page 286 – 287 #1, 3, 5-7, 11, 12, 25, 27, 37, 38, 40

CALCULUS CHAPTER 5 NOTES

SECTION 5-5 Trapezoidal and Simpson's Rule

Using Trapezoids to estimate the area under a curve.



Trapezoidal Rule:

$$\boldsymbol{T = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + 2y_3 + y_4)\ where\ h = \frac{b-a}{n}}$$

Or:

$$\boldsymbol{T = \frac{+}{2}}$$

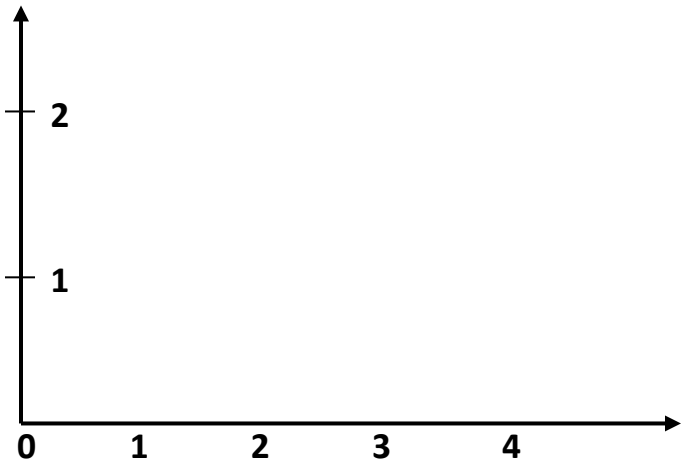
Relationship between Concavity and the Trapezoidal Rule:

Simpson's Rule:

$$S = \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$$

Example: Use the a.) Trapezoidal Rule and b.) Simpson's Rule to approximate the area using n = 4 subintervals.

$$\int_0^4 \sqrt{x}\ dx$$



Actual:

$$\int_0^4 \sqrt{x}\ dx$$

ASSIGNMENT: Page 295 – 296 Quick Review #1, 5, 6, 7; Exercises #1 – 3, 9 a&b

CALCULUS CHAPTER FIVE ASSIGNMENTS

SECTION 5-1 AREAS UNDER CURVES

ASSIGNMENT: Page 254 – 255 # 1, 2, 10, 12

SECTION 5-2 REIMANN SUMS

ASSIGNMENT: Page 267 Quick review #1, 2, 4 - 8; Exercises #1-12, 29-32

SECTION 5-3 DEFINITE INTEGRALS

ASSIGNMENT: Page 274 – 275 #1 - 12

SECTION 5-3 (Day 2) AVERAGE VALUE

ASSIGNMENT: Page 275 #13 – 16, 20, 21 (crosses at x = 2), 22 (crosses at x = 1)

SECTION 5-4 1st FUNDAMENTAL THEOREM OF CALCULUS

ASSIGNMENT: Page 286 – 287 #1, 3, 5-7, 11, 12, 25, 27, 37, 38, 40

SECTION 5-5 TRAPEZOIDAL AND SIMPSON'S RULE

ASSIGNMENT: Page 295 – 296 Quick Review #1, 5, 6, 7; Exercises #1 – 3, 9 a&b

CHAPTER FIVE REVIEW SHEET

CHAPTER FIVE REVIEW SHEET

CHAPTER FIVE TEST