

# ALGEBRA 2 CHAPTER 6 NOTES

## SECTION 6-1 POLYNOMIALS

### Objectives:

Identify, evaluate, add, and subtract polynomials. CC.9-12.F.IF.7c; CC.9-12.A.APR.1

Classify and graph polynomials. CC.9-12.A.CED.2; CC.9-12.A.CED.2

A \_\_\_\_\_ is a number or a product of numbers and variables with whole number exponents.

A \_\_\_\_\_ is a monomial or a sum or difference of monomials. Each monomial in a polynomial is a term.

Polynomials have no variables in \_\_\_\_\_, no roots or absolute values of variables, and all variables have \_\_\_\_\_.

Polynomials:

Not polynomials:

The \_\_\_\_\_ is the sum of the exponents of the variables.

Identify the degree of each monomial.

A.  $z^6$

B.  $5.6$

C.  $8xy^3$

D.  $a^2bc^3$

An \_\_\_\_\_ is given by the term with the greatest degree.

A \_\_\_\_\_ is the coefficient of the first term.

Standard Form

Leading coefficient      Degree of polynomial

$5x^3 + 8x^2 + 3x - 17$

Degree of term:    3                    2                    1                    0

A polynomial with two terms is called a \_\_\_\_\_, and a polynomial with three terms is called a \_\_\_\_\_.

A polynomial can also be classified by its \_\_\_\_\_.

Classifying Polynomials by Degree		
Name	Degree	Example
Constant	0	$-9$
Linear	1	$x - 4$
Quadratic	2	$x^2 + 3x - 1$
Cubic	3	$x^3 + 2x^2 + x + 1$
Quartic	4	$2x^4 + x^3 + 3x^2 + 4x - 1$
Quintic	5	$7x^5 + x^4 - x^3 + 3x^2 + 2x - 1$

Rewrite each polynomial in standard form. Then identify the leading coefficient, degree, and number of terms. Name the polynomial.

A.  $3 - 5x^2 + 4x$

B.  $3x^2 - 4 + 8x^4$

Add or subtract. Write your answer in standard form.

A.  $(2x^3 + 9 - x) + (5x^2 + 4 + 7x + x^3)$

B.  $(3 - 2x^2) - (x^2 + 6 - x)$

Graph each polynomial function on a calculator. Describe the graph and identify the number of real zeros.

A.  $f(x) = 2x^3 - 3x$

B.  $f(x) = -\frac{1}{6}x^4 + 2x^2 - 2$

**ALGEBRA 2 CHAPTER 6 NOTES**  
**SECTION 6-2 MULTIPLYING POLYNOMIALS**

**Objectives:**

**Multiply polynomials.**

**Use binomial expansion to expand binomial expressions that are raised to positive integer powers. *CC.9-12.A.APR.5; CC.9-12.A.APR.1; CC.9-12.A.APR.4***

**Find each product.**

**A.  $4y^2(y^2 + 3)$**

**B.  $fg(f^4 + 2f^3g - 3f^2g^2 + fg^3)$**

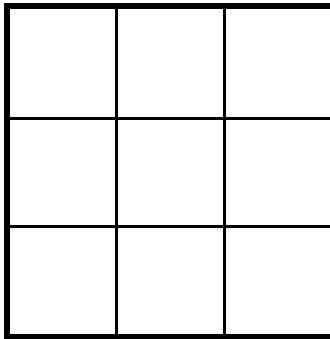
**C.  $3cd^2(4c^2d - 6cd + 14cd^2)$**

**Find the product. Using “rainbows”**

**$(a - 3)(2 - 5a + a^2)$**

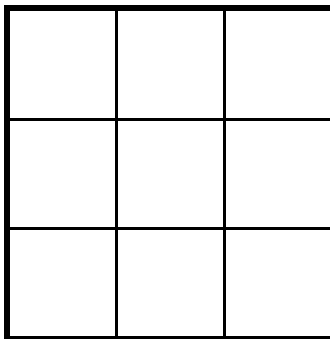
**Find the product.**

**$(y^2 - 7y + 5)(y^2 - y - 3)$**



**Find the product.**

**$(x^2 - 4x + 1)(x^2 + 5x - 2)$**



**Pascal's triangle.**

**Binomial Theorem**

**Expand:**

$$(k - 5)^3$$

$$(3x + 1)^4$$

**ALGEBRA 2 CHAPTER 6 NOTES**  
**SECTION 6-3 DIVIDING POLYNOMIALS**

**Objectives:**

Use long division and synthetic division to divide polynomials. CC.9-12.A.APR.2; CC.9-12.A.APR.6

Divide using long division.

$$(2y^2 + 2y^3 + 25) \div (y - 3)$$

$$y - 3 \overline{) 2y^3 - y^2 + 0y + 25}$$

$$(15x^2 + 8x - 12) \div (3x + 1)$$

$$3x + 1 \overline{) 15x^2 + 8x - 12}$$

\_\_\_\_\_ is a shorthand method of dividing a polynomial by a linear binomial by using only the coefficients.

$$(3x^4 - x^3 + 5x - 1) \div (x + 2)$$

You can use synthetic division to evaluate polynomials. This process is called \_\_\_\_\_.

$$P(x) = 2x^3 + 5x^2 - x + 7 \text{ for } x = 2$$

$$P(x) = x^3 + 3x^2 + 4 \text{ for } x = -3$$



**ALGEBRA 2 CHAPTER 6 NOTES**  
**SECTION 6-4 FACTORING POLYNOMIALS**

**Objectives:**

Use the Factor Theorem to determine factors of a polynomial.

CC.9-12.A.APR.2; CC.9-12.A.APR.3;

Factor the sum and difference of two cubes. CC.9-12.A.APR.4; CC.9-12.A.SSE.2

The \_\_\_\_\_ states that if a polynomial is divided by  $(x - a)$ , the remainder will be a \_\_\_\_\_.

The \_\_\_\_\_: if the remainder is 0, then  $(x - a)$  is a \_\_\_\_\_.

Determine whether the given binomial is a factor of the polynomial  $P(x)$ .

A.  $(x + 1); (x^2 - 3x + 1)$

B.  $(x + 2); (3x^4 + 6x^3 - 5x - 10)$

Factor by grouping:

$$x^3 - x^2 - 25x + 25.$$

$$2x^3 + x^2 + 8x + 4$$

**Factoring the Sum and the Difference of Two Cubes**

METHOD	ALGEBRA
Sum of two cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Difference of two cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$4x^4 + 108x$$

$$125d^3 - 8$$





ALGEBRA 2 CHAPTER 6 NOTES  
SECTION 6-5 FINDING REAL ROOTS

Objectives:

Identify the multiplicity of roots.

CC.9-12.A.APR.3

Use the Rational Root Theorem and the irrational Root Theorem to solve polynomial equations. CC.9-12.A.REI.11; CC.9-12.A.CED.1

Solve the polynomial equation by factoring.

$$4x^5 + 4x^4 - 24x^3 = 0$$

$$x^4 + 25 = 26x^2$$

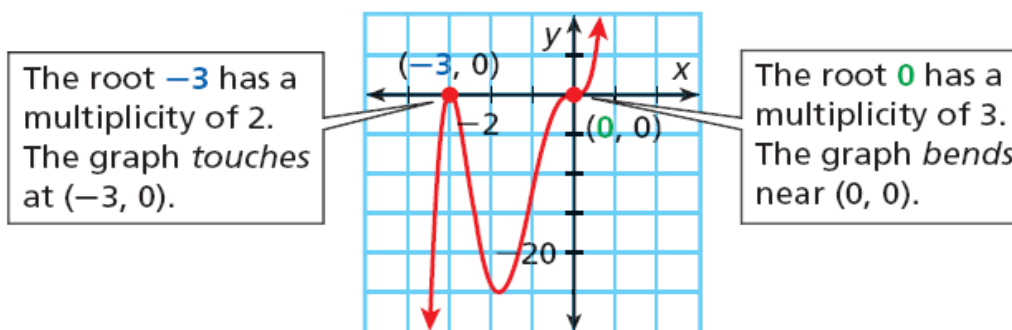
Sometimes a polynomial equation has a factor that appears more than once. This creates a \_\_\_\_\_

The \_\_\_\_\_ of root  $r$  is the number of times that  $x - r$  is a factor of  $P(x)$ .

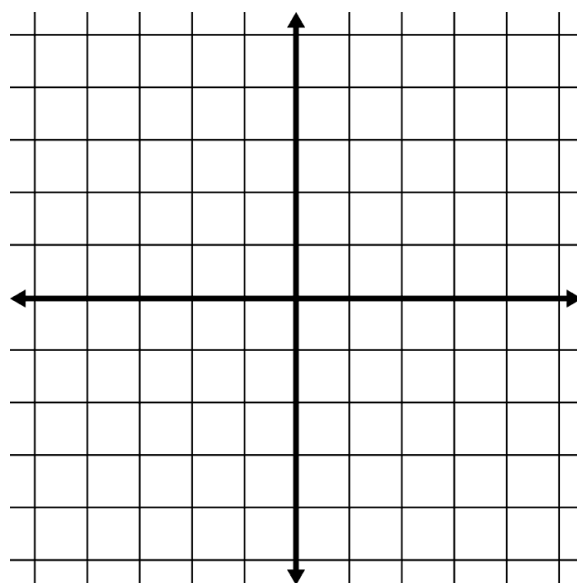
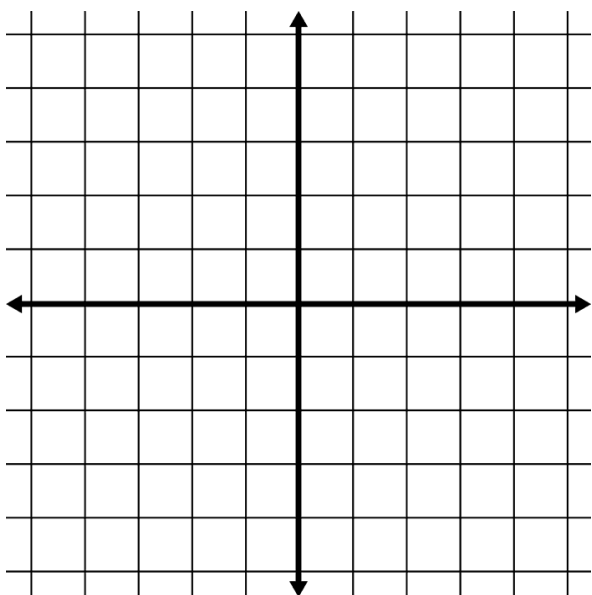
When a real root has a multiplicity of one, the graph of  $y = P(x)$  will \_\_\_\_\_ the  $x$ -axis cleanly.

When a real root has \_\_\_\_\_ multiplicity, the graph of  $y = P(x)$  \_\_\_\_\_ the  $x$ -axis but \_\_\_\_\_.

When a real root has \_\_\_\_\_ multiplicity greater than 1, the graph \_\_\_\_\_ or "swooshes" as it crosses the  $x$ -axis.



Looking at the multiplicities of the previous equations, sketch a rough graph:



\_\_\_\_\_ can help you find all possible rational roots of a polynomial equation.

### **Rational Root Theorem**

If the polynomial  $P(x)$  has integer coefficients, then every rational root of the polynomial equation  $P(x) = 0$  can be written in the form  $\frac{p}{q}$ , where  $p$  is a factor of the constant term of  $P(x)$  and  $q$  is a factor of the leading coefficient of  $P(x)$ .

The \_\_\_\_\_ say that irrational roots come in \_\_\_\_\_. For example, if you know that  $1 + \sqrt{2}$  is a root of  $x^3 - x^2 - 3x - 1 = 0$ , then you know that  $1 - \sqrt{2}$  is also a root.

Identify all the real roots of

$$2x^3 - 3x^2 - 10x - 4 = 0.$$

$$3x^4 - 7x^2 + 6x - 12 = 0$$

**ALGEBRA 2 CHAPTER 6 NOTES**  
**SECTION 6-6 FUNDAMENTAL THEOREM OF ALGEBRA**

**Objectives:**

Use the Fundamental Theorem of Algebra and its corollary to write a polynomial equation of least degree with given roots. (CC.9-12.N.CN7,CN8,CN90)

Identify all of the roots of a polynomial equation.(CC.9-12.A.APR.2)

Also (CC.9-12.A.CED1, CC.9-12.A.REI.11)

**3 other names for a root:**

**Write the simplest polynomial with roots  $-1$ ,  $2/3$ , and  $4$ .**

**Write the simplest function with zeros  $2 + i$ ,  $2 - i$ , and  $1$ .**

**THE FUNDAMENTAL THEOREM OF ALGEBRA**

**Solve  $x^4 - 3x^3 + 5x^2 - 27x - 36 = 0$  by finding all roots.**

**Solve  $x^4 + 4x^3 - x^2 + 16x - 20 = 0$  by finding all roots.**

**Write the simplest function with zeros  $2i$ ,  $2 - i$ , and  $3$ .**



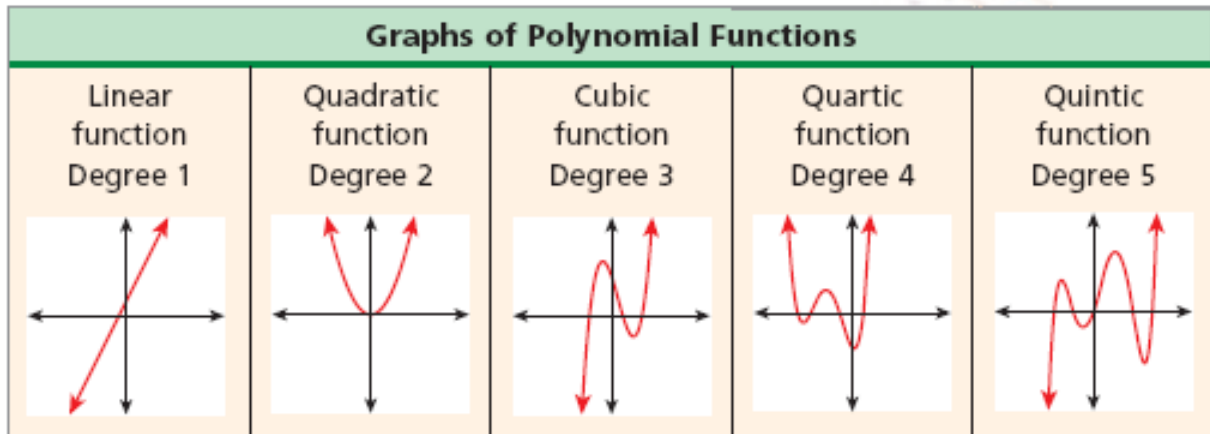
## ALGEBRA 2 CHAPTER 6 NOTES

### SECTION 6-7 GRAPHS OF POLYNOMIALS

Objectives:

Use properties of end behavior to analyze, describe, and graph polynomial functions. CC.9-12.A.APR.3; CC.9-12.F.IF.7c

Identify and use maxima and minima of polynomial functions to solve problems. CC.9-12.A.CED.2; CC.9-12.A.CED.3



\_\_\_\_\_ is a description of the values of the function as  $x$  approaches infinity \_\_\_\_\_ or negative infinity \_\_\_\_\_.

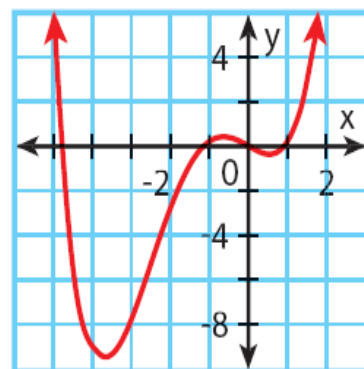
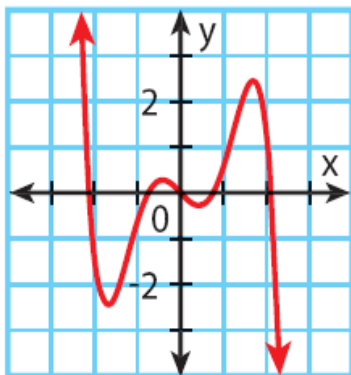
The \_\_\_\_\_ coefficient of a polynomial function determine its end behavior.

Identify the leading coefficient, degree, and end behavior.

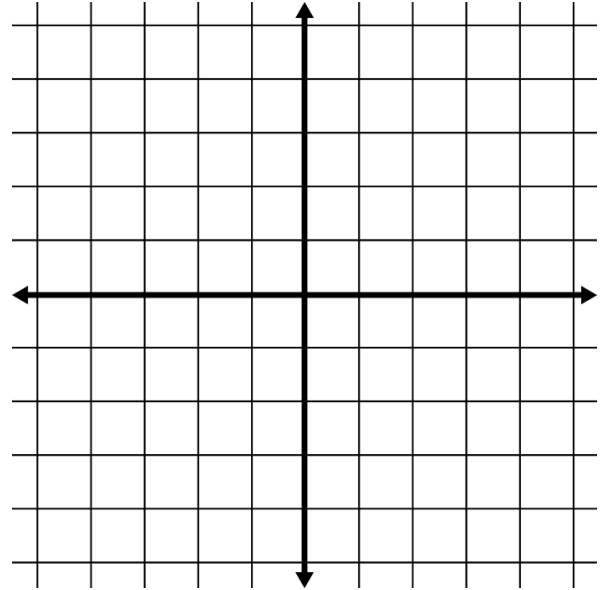
A.  $Q(x) = -x^4 + 6x^3 - x + 9$

B.  $P(x) = 2x^5 + 6x^4 - x + 4$

Determine a possible equation given the graph.



Graph the function.  $f(x) = x^3 + 4x^2 + x - 6$ .



A \_\_\_\_\_ is where a graph changes from \_\_\_\_\_.

A turning point corresponds to a *local* \_\_\_\_\_.

**ALGEBRA 2 CHAPTER 6 NOTES**  
**SECTION 6-8 TRANSFORMING POLYNOMIALS**

Objectives:

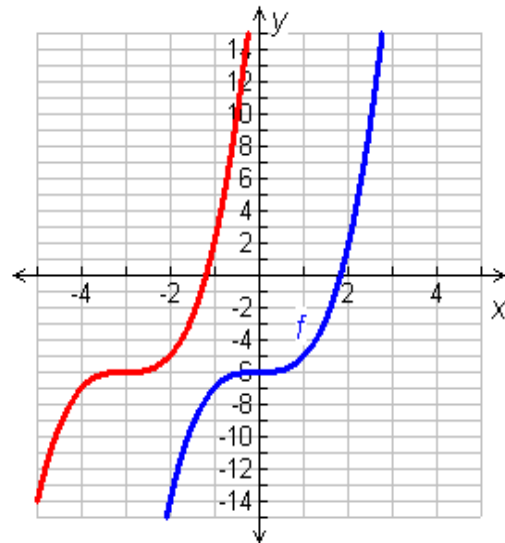
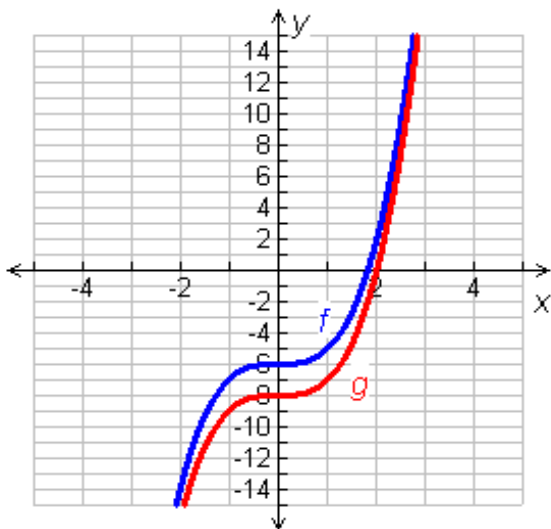
Transform polynomial functions.

CC.9-12.F.IF.7c; CC.9-12.F.BF.3;

CC.9-12.A.CED.3

Transformations of $f(x)$		
Transformation	$f(x)$ Notation	Examples
Vertical translation	$f(x) + k$	$g(x) = x^3 + 3$ 3 units up $g(x) = x^3 - 4$ 4 units down
Horizontal translation	$f(x - h)$	$g(x) = (x - 2)^3$ 2 units right $g(x) = (x + 1)^3$ 1 unit left
Vertical stretch/ compression	$af(x)$	$g(x) = 6x^3$ stretch by 6 $g(x) = \frac{1}{2}x^3$ compression by $\frac{1}{2}$
Horizontal stretch/ compression	$f\left(\frac{1}{b}x\right)$	$g(x) = \left(\frac{1}{5}x\right)^3$ stretch by 5 $g(x) = (3x)^3$ compression by $\frac{1}{3}$
Reflection	$-f(x)$ $f(-x)$	$g(x) = -x^3$ across $x$ -axis $g(x) = (-x)^3$ across $y$ -axis

For  $f(x) = x^3 - 6$ , write the rule for each function and sketch its graph.

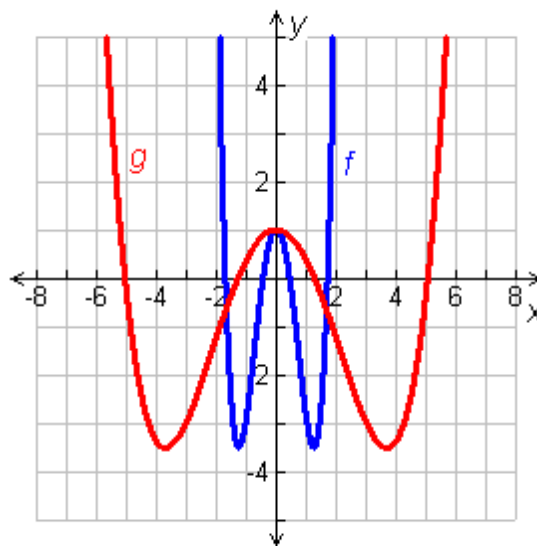
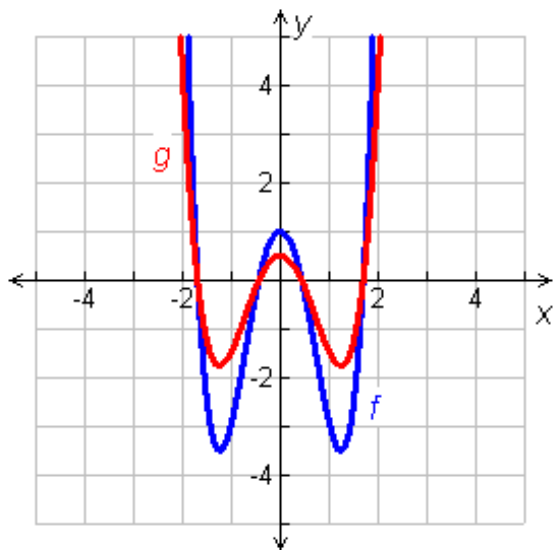


Let  $f(x) = x^3 + 5x^2 - 8x + 1$ . Write a function  $g$  that performs each transformation.

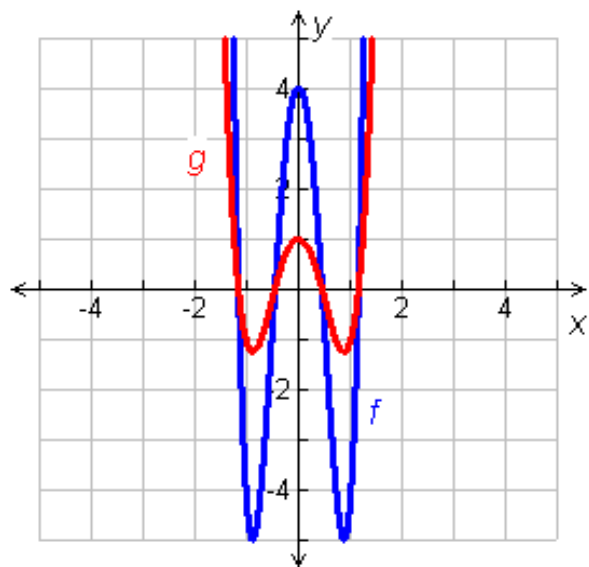
Reflect  $f(x)$  across the  $x$ -axis.

Reflect  $f(x)$  across the  $y$ -axis.

Let  $f(x) = 2x^4 - 6x^2 + 1$ . Graph  $f$  and  $g$  on the same coordinate plane. Describe  $g$  as a transformation of  $f$ .



Let  $f(x) = 16x^4 - 24x^2 + 4$ . Graph  $f$  and  $g$  on the same coordinate plane. Describe  $g$  as a transformation of  $f$ .



Write a function that transforms  $f(x) = 6x^3 - 3$  in each of the following ways.  
Compress vertically by a factor of  $1/3$ , and shift 2 units right.

Reflect across the  $y$ -axis and shift 2 units down.

Write a function that transforms  $f(x) = 8x^3 - 2$  in each of the following ways.  
Compress vertically by a factor of  $1/2$ , and move the  $x$ -intercept 3 units right.



## ALGEBRA 2 CHAPTER 6 NOTES

### SECTION 6-9 CURVE FITTING

Objectives:

Use finite differences to determine the degree of a polynomial that will fit a given set of data. CC.9-12.F.IF.7c; CC.9-12.A.CED.3; CC.9-12.A.CED.2

Use technology to find polynomial models for a given set of data.

Finite Differences of Polynomials		
Function Type	Degree	Constant Finite Differences
Linear	1	First
Quadratic	2	Second
Cubic	3	Third
Quartic	4	Fourth
Quintic	5	Fifth

Use finite differences to determine the degree of the polynomial that best describes the data.

<b>x</b>	-6	-3	0	3	6	9
<b>y</b>	-9	16	26	41	78	151

<b>x</b>	12	15	18	21	24	27
<b>y</b>	3	23	29	29	31	43

The table below shows the population of a city from 1960 to 2000. Write a polynomial function for the data.

<b>Year</b>	1960	1970	1980	1990	2000
<b>Population (Thousands)</b>	4,267	5,185	6,166	7,830	10,812

The table below shows the gas consumption of a compact car driven a constant distance at various speed. Write a polynomial function for the data

<b>Speed</b>	25	30	35	40	45	50	55	60
<b>Gas (gal)</b>	23.8	25	25.2	25	25.4	27	30.6	37

Often, real-world data can be too \_\_\_\_\_ for you to use finite differences or find a polynomial function that fits perfectly. In these situations, you can use the regression feature of your graphing calculator. Remember that the closer the  $R^2$ -value is to \_\_\_\_, the \_\_\_\_\_.

The table below shows the opening value of a stock index on the first day of trading in various years. Use a polynomial model to estimate the value on the first day of trading in 2000

<b>Year</b>	1994	1995	1996	1997	1998	1999
<b>Price (\$)</b>	683	652	948	1306	863	901

The table below shows the opening value of a stock index on the first day of trading in various years. Use a polynomial model to estimate the value on the first day of trading in 1999.

<b>Year</b>	1994	1995	1996	2000	2003	2004
<b>Price (\$)</b>	3754	3835	5117	11,497	8342	10,454